Utah Core Standards Benchmarks
Mathematics

Test Name	Form	Number of Items	Standard	Description of the Standard
Benchmark Module: Mathematics Secondary Mathematics I - Algebra	A	10	MI.A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.
			MI.A.CED.3	Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
			MI.A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
			MI.A.REI.12	Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corre sponding half-planes.
			MI.A.REI.3	Solve equations and inequalities in one variable. **a.** Solve one-variable equations and literal equations to highlight a variable of interest. **b.** Solve compound inequalities in one variable, including absolute value inequalities. **c.** Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). For example, $5^x = 125$ or $2^x = 1/16$.
			MI.A.REI.6	Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.
	B	11	MI.A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and simple exponential functions.
			MI.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

			MI.A.REI.12	Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corre sponding half-planes.
			MI.A.REI.3	Solve equations and inequalities in one variable. **a.** Solve one-variable equations and literal equations to highlight a variable of interest. **b.** Solve compound inequalities in one variable, including absolute value inequalities. **c.** Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). For example, $5^x = 125$ or $2^x = 1/16$.
			MI.A.REI.6	Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.
	C	**12**	**MI.A.CED.2**	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
			MI.A.REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
			MI.A.REI.12	Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corre sponding half-planes.
			MI.A.REI.3	Solve equations and inequalities in one variable. **a.** Solve one-variable equations and literal equations to highlight a variable of interest. **b.** Solve compound inequalities in one variable, including absolute value inequalities. **c.** Solve simple exponential equations that rely only on application of the laws of exponents (limit solving exponential equations to those that can be solved without logarithms). For example, $5^x = 125$ or $2^x = 1/16$.
			MI.A.REI.6	Solve systems of linear equations exactly and approximately (numerically, algebraically, graphically), focusing on pairs of linear equations in two variables.

			MI.A.SSE.1a	Interpret linear expressions and exponential expressions with integer exponents that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
Benchmark Module: Mathematics Secondry Mathematics I - Geometry	**A**	**10**	MI.G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
			MI.G.CO.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
			MI.G.CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
			MI.G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
			MI.G.CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide whether they are congruent.
			MI.G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

			MI.G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
			MI.G.GPE.5	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
	B	**9**	**MI.G.CO.12**	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
			MI.G.CO.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
			MI.G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
			MI.G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
			MI.G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
			MI.G.GPE.5	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

			MI.G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.
	C	**10**	MI.G.CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Emphasize the ability to formalize and defend how these constructions result in the desired objects. *For example, copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
			MI.G.CO.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
			MI.G.CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Point out the basis of rigid motions in geometric concepts, for example, translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.
			MI.G.CO.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
			MI.G.CO.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
			MI.G.GPE.5	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
			MI.G.GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles; connect with The Pythagorean Theorem and the distance formula.

Benchmark Module: Mathematics Secondary Mathematics I - Number Quantity/Functions/Statistics and Probability	**A**	**23**	**MI.F.BF.1a**	Write a function that describes a relationship between two quantities. **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
			MI.F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.
			MI.F.IF.1	Understand that a function from one set (called the domain) to anotherset (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x* . The graph of f is the graph of the equation *y = f(x)* .
			MI.F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
			MI.F.IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*
			MI.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
			MI.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
			MI.F.IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **a.** Graph linear functions and show intercepts.

			MI.F.LE.1c	Distinguish between situations that can be modeled with linear functions and with exponential functions. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
			MI.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
			MI.N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
			MI.N.Q.2	Define appropriate quantities for the purpose of descriptive modeling.
			MI.S.ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.
			MI.S.ID.6c	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. **c.** Fit a linear function for scatter plots that suggest a linear association.
			MI.S.ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
	B	**23**	MI.F.BF.1a	Write a function that describes a relationship between two quantities. **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
			MI.F.BF.3	Identify the effect on the graph of replacing *f(x) by f(x) + k* , for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Relate the vertical translation of a linear function to its y-intercept. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
			MI.F.IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

			MI.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*
			MI.F.IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **a.** Graph linear functions and show intercepts.
			MI.F.IF.9	Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example,* *compare the growth of two linear functions, or two exponential functions such as* $y=3^n$ *and* $y=100\bullet 2^n$.
			MI.F.LE.1c	Distinguish between situations that can be modeled with linear functions and with exponential functions. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
			MI.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
			MI.N.Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
			MI.N.Q.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
			MI.S.ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
			MI.S.ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Calculate the weighted average of a distribution and interpret it as a measure of center.

Benchmark Module: Mathematics Secondary Mathematics II - Functions	**A**	**8**	**MII.F.IF.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior
			MII.F.IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **a.** Graph linear and quadratic functions and show intercepts, maxima, and minima.
			MII.F.IF.9	Compare properties of two functions, each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
			MII.F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Compare linear and exponential growth to quadratic growth.
			MII.F.TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.
	B	**8**	**MII.F.IF.4**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior
			MII.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

			MII.F.IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **a.** Graph linear and quadratic functions and show intercepts, maxima, and minima.
			MII.F.IF.8a	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
			MII.F.TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle.
Benchmark Module: Mathematics Secondary Mathematics II - Geometry	**A**	**11**	**MII.G.C.3**	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
			MII.G.C.4	Construct a tangent line from a point outside a given circle to the circle.
			MII.G.CO.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
			MII.G.CO.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
			MII.G.CO.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

			MII.G.GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Informal arguments for area formulas can make use of the way in which area scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
			MII.G.SRT.1b	Verify experimentally the properties of dilations given by a center and a scale factor. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
			MII.G.SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
	B	**12**	**MII.G.C.4**	Construct a tangent line from a point outside a given circle to the circle.
			MII.G.C.5	Derive, using similarity, the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
			MII.G.CO.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
			MII.G.CO.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
			MII.G.CO.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

			MII.G.GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Informal arguments for volume formulas can make use of the way in which volume scale under similarity transformations: when one figure results from another by applying a similarity transformation, volumes of solid figures scale by k^3 under a similarity transformation with scale factor *k*
			MII.G.SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
			MII.G.SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Benchmark Module: Mathematics Secondary Mathematics II - Number & Quantity/Algebra	**A**	**19**	MII.A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
			MII.A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
			MII.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
			MII.A.REI.4a	Solve quadratic equations in one variable. **a.** Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
			MII.A.REI.4b	Solve quadratic equations in one variable. **b.** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b* .
			MII.A.SSE.1a	Interpret quadratic and exponential expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
			MII.A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

			MII.N.CN.1	Know there is a complex number i such that i^2 = -1, and every complex number has the form *a + bi* with *a* and *b* real.
			MII.N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.
			MII.N.CN.8	Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
			MII.N.CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
			MII.N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
	B	**20**	MII.A.APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
			MII.A.CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
			MII.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
			MII.A.REI.4b	Solve quadratic equations in one variable. **b.** Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a ± bi* for real numbers *a* and *b* .
			MII.A.SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.
			MII.N.CN.1	Know there is a complex number i such that i^2 = -1, and every complex number has the form *a + bi* with *a* and *b* real.
			MII.N.CN.7	Solve quadratic equations with real coefficients that have complex solutions.
			MII.N.CN.8	Extend polynomial identities to the complex numbers. Limit to quadratics with real coefficients. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
			MII.N.CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

			MII.N.RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
			MII.N.RN.3	Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations (e.g., finding the perimeter of a square of area 2).
Benchmark Module: Mathematics Secondary Mathematics III - Functions	**A**	**12**	**MIII.F.BF.1b**	Write a function that describes a relationship between two quantities. **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
			MIII.F.BF.4a	Find inverse functions. **a.** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. Include linear, quadratic, exponential, logarithmic, rational, square root, and cube root functions. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
			MIII.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
			MIII.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
			MIII.F.IF.7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.

			MIII.F.IF.7c	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
			MIII.F.IF.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
			MIII.F.LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e* ; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log *xy = log x + log y.*
	B	**11**	**MIII.F.BF.1b**	Write a function that describes a relationship between two quantities. **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
			MIII.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
			MIII.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h*(n) gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*

			MIII.F.IF.7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.
			MIII.F.IF.7c	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **c.** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
			MIII.F.IF.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
			MIII.F.LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e* ; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log *xy = log x + log y.*
	C	**11**	MIII.F.BF.1b	Write a function that describes a relationship between two quantities. **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
			MIII.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

			MIII.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* h*(n) gives the number of person-hours it takes to assemble* n *engines in a factory, then the positive integers would be an appropriate domain for the function.*
			MIII.F.IF.7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast square root, cubed root, and step functions with all other functions.
			MIII.F.IF.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. **e.** Graph exponential and logarithmic functions, showing intercepts and end behavior; and trigonometric functions, showing period, midline, and amplitude.
			MIII.F.LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a, c,* and *d* are numbers and the base *b* is 2, 10, or *e* ; evaluate the logarithm using technology. Include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log *xy = log x + log y.*
Benchmark Module: Mathematics Secondary Mathematics III - Number & Quantity/Algebra	A	14	MIII.A.APR.1	Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
			MIII.A.APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
			MIII.A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n* , where *x* and *y* are any numbers. *For example, with coefficients determined by Pascal's Triangle.*

		MIII.A.APR.6	Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)* , where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x),* using inspection, long division or, for the more complicated examples, a computer algebra system.
		MIII.A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
		MIII.A.CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
		MIII.A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
		MIII.A.SSE.1a	Interpret polynomial and rational expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
		MIII.A.SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*
		MIII.N.CN.8	Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$*.*
B	**14**	MIII.A.APR.1	Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
		MIII.A.APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
		MIII.A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n* , where *x* and y are any numbers. *For example, with coefficients determined by Pascal's Triangle.*

			MIII.A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
			MIII.A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+ r/n)^{nt}$
			MIII.A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
			MIII.A.SSE.1a	Interpret polynomial and rational expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
			MIII.A.SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
			MIII.N.CN.8	Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
			MIII.N.CN.9	Know the Fundamental Theorem of Algebra; show that it is true for quadraticpolynomials. Limit to polynomials with real coefficients.
	C	**14**	**MIII.A.APR.1**	Understand that all polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
			MIII.A.APR.5	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of *x* and *y* for a positive integer *n* , where *x* and *y* are any numbers. *For example, with coefficients determined by Pascal's Triangle.*
			MIII.A.APR.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
			MIII.A.CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange the compound interest formula to solve for* t: $A = P(1+ r/n)^{nt}$

			MIII.A.REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
			MIII.A.SSE.1a	Interpret polynomial and rational expressions that represent a quantity in terms of its context. **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
			MIII.A.SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$*.*
			MIII.N.CN.8	Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$*.*
Benchmark Module: Mathematics Secondary Mathematics III - Trigonomic Functions/Geometry	**A**	**10**	MIII.F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
			MIII.F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
			MIII.F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
			MIII.G.GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
			MIII.G.MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
			MIII.G.MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
	B	**11**	MIII.F.TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

			MIII.F.TF.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π6, and use the unit circle to express the values of sine, cosine, and tangent for π – x, π + x, and 2π– x in terms of their values for x, where x is any real number.
			MIII.F.TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
			MIII.G.GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
			MIII.G.MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
			MIII.G.MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
Benchmark Module: Mathematics Secondary Mathematics III - Statistics & Probability	**A**	**8**	**MIII.S.IC.1**	Understand that statistics allow inferences to be made about populationparameters based on a random sample from that population.
			MIII.S.IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
			MIII.S.ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.